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TRANSFER OF HEAT AND WATER ACROSS THE SEA SURFACE

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by R.B.M N.Y.U. Oct. 30, 1941

General discussion

Next to the sea surface there is usually a layer of water which is nearly isothermal vertically. The thickness of this layer is extremely variable; in winter it may exceed 100 meters, while in summer it is often less than a centimeter thick. The temperature range vertically in these layers is not more than a few tenths of one degree Fahrenheit.

A nearly isothermal layer may be produced and deepened by stirring due to wind friction, and in shallow water sometimes bottom friction mixes the water from bottom to top. When a nearly isothermal layer exists it is deepened by horizontal convergence of the layer, as happens when the wind blows the surface water against the shore. Conversely, horizontal divergence of surface water may result in the thinning or destruction of the isothermal layer.

The other class of factors which tend to form, deepen or destroy the nearly isothermal layer is the one including all effects which change the density of the water. The only important factors of this class are those involving the transfer of water and heat across the sea surface. These are solar radiation, nocturnal radiation, conduction to or from the atmosphere, evaporation, and precipitation.

Of these all but solar radiation affect directly only the very surface film, a layer less than a millimeter thick. Their effect on the water below is by means of conduction, diffusion and mixing in the water. Conduction and diffusion are negligibly effective over distances as great as a few centimeters, so mixing is necessary for penetration to greater depths.

By nocturnal radiation is meant the radiation from the surface to the atmosphere and to space, less the radiation from the atmosphere to the surface. It is operative at all times, but its name comes from the fact that at night, when solar radiation is absent, it accounts for the total radiation transfer at the surface. This is long-wave radiation, for which water is very opaque. On the other hand a large part of the energy of solar radiation is in the visible wave lengths, for which water is relatively transparent. Of the solar radiation absorbed by the sea at least 5% is absorbed in the first tenth of a millimeter, 14% in the first millimeter, 27% in the first centimeter and 45% in the first 10 centimeters; in the first meter 64% to 90% is absorbed.+

Solar radiation adds heat to the surface water. Evaporation (except when it is negative: condensation on the sea surface) takes heat from it, as does usually nocturnal radiation. Conduction between the air and the water may be in either direction. The net result of these factors determines whether the water is gaining or losing heat.

The effect on the vertical temperature gradient of heating or cooling the surface water depends on the water's stability, or increase per unit length downward of density. Stability can have a wide range of positive values, but it can be only slightly negative (except in a very thin layer). If there is negative stability, or instability, in a layer, convection occurs, mixing the water in the layer and maintaining the instability at a low value. Thus there are essentially two types of layer, the one of nearly zero stability, or of nearly neutral equilibrium, and the stable layer. In a stable

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+ Dorsey, Properties of ordinary water-substance, pp. 333-334.

layer mixing is inhibited so that a large temperature gradient can be produced by the heat exchange across the sea surface. A layer of nearly neutral equilibrium is easily mixed by friction, and, if it is on the unstable side of neutral equilibrium, it is mixed by convection, so that practically always it is nearly isothermal. Such a layer on the unstable side of neutral is called a convection layer.

Since density is a function of temperature and salinity, the stability of a layer is affected by any change of temperature or salinity which is not evenly distributed through the layer. Solar radiation acting alone, since its heating effect decreases from the sea surface less and less steeply to practically zero in a very few fathoms, would produce stability which decreases from a maximum value at the surface. If the resulting temperature and stability distributions were then subjected to evaporation alone, which increases the density of the surface water, a very thin convection layer would immediately form at the surface and then gradually deepen; this layer would be nearly isothermal. Evaporation increases surface density not only by cooling, but also by increasing the salinity. Conversely precipitation tends to produce stability, since rain water regardless of its temperature is always lighter than sea water of the salinities commonly found.

Usually at night the density of the surface water is being increased by nocturnal radiation and evaporation, forming a convection layer which is nearly isothermal. At noontime nocturnal radiation and evaporation are usually outweighted by solar radiation. Nevertheless, since little of the solar radiation is absorbed in the surface film, a thin convection layer often persists at the surface even though the stability and the magnitude of the temperature ...

gradient may be increasing below.

For many purposes one can neglect the details of the equilibrium very near the sea surface, which are affected by the appreciable depth of penetration of solar radiation, so it is only necessary to consider the net effect on the water density of the transfer of heat and water across the surface. In the absence of frictional mixing and of horizontal flow, if this net effect is such as to decrease density, the stability is increased and, if heat is added also, the temperature gradient becomes more negative. However, if this net effect is such as to increase density, the nearly isothermal layer increases in depth. On the other hand, when the surface layer is frictionally stirred with sufficient activity, the water and heat exchange does not alter its nearly neutral equilibrium and its nearly isothermal character.

The effects of solar heating, nocturnal radiation, conduction to or from the atmosphere, and evaporation will be treated individually below. The presentation is aimed to make possible a quantitative evaluation of each from the common meteorological observations obtained on shipboard, especially toward obtaining the net effect of all four. Precipitation is omitted from individual attention because quantitative observations of it are not commonly available and usually it is sufficient to consider only its qualitative effect.

Vapor pressure

The vapor pressure in the air plays an important part both in radiation processes and in evaporation.

By way of introduction it may be recalled that, if a pan of

water is placed in a container covered with insulation and the air completely removed, the system comes finally to a constant temperature and the vapor pressure reaches a value which depends on this temperature alone. This value of vapor pressure for various temperatures is given in Table 1. The unit of pressure employed is the millibar, defined as 1000 dynes per square centimeter. In turn, 1000 mb is a little less than one atmosphere.

In another experiment suppose the pan is covered and placed in the container with dry air, and this time the container is not insulated but placed in a constant temperature bath. If the pressure in the container is measured after its temperature comes to equilibrium, then the cover removed from the pan and the pressure measured after equilibrium is again reached, it is found that the pressure increases by an amount equal to the vapor pressure attained at the same temperature in the first experiment. Furthermore the amount of water evaporated is the same in each case. So in each case the vapor pressure and the density of water vapor is the same.

In the final state of either experiment the vapor is said to be saturated. The vapor pressure attained is thus independant of the presence of air and depends only on the temperature, and this function of temperature, given in Table 1, is called saturation vapor pressure.

In the first experiment the evaporation is almost instantaneous; it is necessary to wait for complete equilibrium only in order that temperature equalize completely throughout vapor and liquid water. In the second experiment the rate of evaporation is slow, because the vapor diffuses slowly through the air. Even in the second experiment, however, the adjustment immediately at the water surface is instantaneous, so that at the interface there is saturation and air and water have the same temperature.

If in these experiments the pan contained salt water, which is hygroscopic, a lower value of vapor pressure would be reached for the same temperature. For average sea water the vapor pressure is about 2 per cent less than for pure water. Vapor pressure over sea water is given in Table 2.

Evaporation involves a loss of heat from the water surface. At 50°F the loss is 590 calories for each gram of water evaporated. This quantity, known as latent heat, varies with temperature only slightly.

One manifestation of this cooling effect is in the so-called wet-bulb temperature, which may be defined as the temperature the air will reach when its vapor becomes saturated without any other gain or loss of heat than the latent heat utilized during the saturation. Air may be brought approximately to its wet-bulb temperature by passing it through wet gauze, as in this case the latent heat required for saturation is supplied by the cooling of the air.

The wet-bulb temperature forms the basis of a common instrument for measuring humidity, the psychrometer. When a small wet object is placed in a stream of unsaturated air it assumes a temperature lower than the air temperature. It loses heat by evaporation and gains heat by conduction from the warmer air. Initially the object cools, but when the loss and gain of heat reach a balance it reaches an equilibrium temperature. In general this temperature depends on the air temperature, humidity and pressure, and also on the shape and size of the object and on the air speed. The equilibrium temperature is however independent of these last factors if the object is about the size of an ordinary thermometer bulb and if the air speed is 7 miles per hour or more. The object then comes very close to the wet-bulb temperature of the air. The psychrometer consists of a wet-bulb

thermometer and a dry-bulb thermometer mounted side by side. If it is reasonably protected from radiation and placed in an air stream of at least 7 miles per hour, the two thermometer readings and the barometric pressure are all the information necessary for computing the humidity. Table 3 gives the formula by which the vapor pressure, e, in the air may be calculated.

Vapor pressure may alternatively be computed from relative humidity and temperature, or vice versa. Relative humidity, f, is defined as the ratio of the actual vapor pressure to the saturation vapor pressure, e_m,

$$f = \frac{e}{e_m} .$$

Radiation laws

The rate at which a body radiates heat, including light, depends on its temperature. Bodies of the same size and at the same temperature but of different substance emit radiation at different rates, but the limiting maximum amount is radiated by a so-called ideal radiator. An ideal radiator furthermore absorbs all radiation falling upon it. The total radiation, E, from an ideal radiator is proportional to the fourth power of its absolute temperature, T, according to Stefan's law,

$$E = sT^4.$$

The constant s has the value 1.178×10^{-7} cal cm^{-2} day^{-1} $(deg\ A)^{-4}$ (calories per square centimeter per day per degree absolute to the fourth power). Many natural bodies approximate closely an ideal radiator. For instance the sun emits practically the same radiation as would an ideal radiator at 5600°A.

The spectral distribution also of the radiation from a ideal radiator depends only on its temperature. The distribution is a

continuous curve with a single maximum, which occurs at a specific wave length for a given temperature. The relationship is given by Wien's displacement law,

(wave length for radiation of maximum intensity) x (absolute temperature) = 2900 microns.

This maximum is at 0.5 micron for solar radiation, at 10 microns for an ideal radiator at 50°F. The micron is a unit of length equivalent to a thousandth of a millimeter. For comparison, the visible band of the spectrum is from 0.4 to 0.7 micron. As the temperature increases, the emission in each wave length increases, but the increase is more rapid in the shorter wave lengths so the maximum shifts to shorter wave lengths.

The intensity of radiation from a body in space varies as the inverse square of the distance from the body. Hence the intensity of solar radiation at the earth's distance from the sun is only about 0.00002 of its intensity at the sun's surface. Since the solar radiation is reduced to this extremely small fraction its spectral energy distribution falls to practically zero except near the peak; only an undetectable amount remains in wave lengths longer than about 3 microns. On the other hand the radiation from an ideal radiator at any atmospheric or oceanic temperature has utterly negligible intensity in wave lengths less than 3 microns. Hence the two spectra are mutually exclusive, so that solar (short-wave) radiation and terrestrial (long-wave) radiation can be treated entirely separately.

The intensity of the radiation in the solar beam in free space at the earth's mean distance from the sun is known in meteorology as the solar constant. Its value is about 2800 cal cm^{-2} day^{-1}.

The albedo of the earth as a whole is about 0.43, which means that 43 per cent of the solar radiation intercepted by the earth is

reflected and scattered back to space. This leaves as the amount of solar radiation absorbed by the atmosphere, ground and ocean an average of 1600 cal cm^{-2} day^{-1} over the area intercepted by the earth. This area is one quarter of the area of the sphere, so an average of 400 cal cm^{-2} day^{-1} is received over the area of the earth's surface. Since the radiation emitted by the earth must practically balance that absorbed, a square centimeter of the earth's surface must radiate an average of 400 calories per day of long-wave radiation, which is the amount emitted by a square centimeter of an ideal radiator at 241°A or -26°F.

The behavior of bodies toward long-wave radiation may be very different than toward short-wave radiation. It has already been mentioned that light penetrates rather far into water, which is very opaque for long-wave radiation. The reflecting power or albedo of clouds and of snow-cover, including most sea-ice, is between 0.7 and 0.8 for solar radiation, but these surfaces are practically ideal radiators for the long-wave radiation they emit at terrestrial temperatures. This is part of the explanation of the lower temperatures of snow or ice surfaces than of bare ground or water, and hence of the greater cooling of the air over them.

The absorption of radiation of a given wave length by a thin layer of semi-transparent material normal to the beam is equal to the absorption coefficient times the amount of radiation entering the layer. It follows for instance that as a narrow wave-length band of solar radiation passes through uniform sea water, the first centimeter of water absorbs a certain fraction of the energy passing through the sea surface, the second centimeter absorbs the same fraction of what is left, and so on. So the first centimeter absorbs more than the second, and the second more than the third. The logarithm of intensity

decreases linearly with distance along the beam, the proportionality factor being the absorption coefficient.

The absorption coefficient of water (or air) varies widely over the spectrum of solar radiation. Since some wave lengths are entirely absorbed in the first centimeter while others are absorbed only slightly, the simple absorption law does not hold for the total intensity. When the absorption coefficient varies over the spectrum of radiation present, each successive layer of equal thickness absorbs a successively lesser fraction of the radiation impinging upon it.

Another concept of importance is Kirchhoff's law. One way of stating this is that for a given wave length, if a body absorbs a certain fraction of the incident radiation, it emits the same fraction of the radiation an ideal radiator at its temperature would emit. Of the radiation emitted by the ocean surface -- the continuous spectrum of an ideal radiator -- certain bands are strongly absorbed by the atmosphere, for certain other bands a cloudless atmsophere is transparent. From Kirchhoff's law it is apparent that the atmosphere radiates back to the ocean in the bands where it absorbs, but gives off no radiation in the bands where it is transparent. The atmosphere is never enough warmer than the water to radiate in its limited spectrum so much energy as is in the complete spectrum of the ocean surface. Hence in cloudless weather the nocturnal radiation over water is always positive: the water radiates more energy than it receives from the atmosphere.

Clouds emit the continuous spectrum of an ideal radiator. If the sky is completely covered with stratus clouds at the same temperature as the sea surface, the clouds send down just as much radiation as the water sends up. Since the clouds are usually colder than the

water, nocturnal radiation is usually positive even in cloudy weather; but occasionally the sky is overcast with clouds sufficiently warm and low so that the nocturnal radiation is negative.

The following supplementary reading is recommended:

Admiralty weather manual, Chapter V.

Petterssen, Introduction to meteorology, Chapter VI.

Haurwitz, Dynamic meteorology, Chapter V.

Brunt, Physical and dynamical meteorology, Chapters VI and VII.

Shaw, Manual of meteorology, Vol.III, Chapters IV and V.

Solar radiation

With the sun in the zenith the intensity of solar radiation received by unit horizontal surface outside the atmosphere is equal to the solar constant if the earth is at its mean distance from the sun. At the equinoxes such a point on the equator receives radiation at an average rate over 24 hours which is $1/\pi$ of the noon intensity, so that one square centimeter receives about 900 calories during the day. In general the radiation received during a day by a horizontal surface outside the atmsophere depends on the latitude and on the time of year ; it is zero inside the arctic circle in midwinter and reaches a maximum of 1150 cal cm^{-2} at the south pole in midsummer. In midsummer at the north pole the value is slightly less, 1080 cal cm^{-2}, because the earth is then further from the sun. This difference between the northern and southern hemispheres at corresponding seasons will be neglected in what follows.

In passing through the atmosphere the intensity of the solar beam is decreased by absorption, scattering, and reflection from clouds At first a cloudless atmosphere will be considered.

The absorption increases as the altitude of the sun decreases. With the sun in the zenith the solar beam passes through one "atmosphere", with the sun at an altitude of 30° it passes through 2 "atmospheres",at 19° through three, at 14° through four, etc. Thus the proportion of solar radiation absorbed in the atmosphere varies throughout the day; for equally transparent air the proportion absorbed is least at noon and becomes very great when the sun is near the horizon.

The depletion of solar radiation in the atmosphere is due mostly to the presence of water vapor and dust. With dust-free air and with the sun in the zenith the proportion of solar radiation lost in transmission to the sea surface varies between about 20 and 35 per cent. For various altitudes of the sun and for two specified amounts of water vapor in the atmosphere the amounts of solar radiation received on a horizontal surface are given in Table 4.

For present purposes the quantity usually desired is the total solar radiation received at the sea surface on a given day at a given spot. Since the instantaneous values of Table 4 are laborious to integrate throughout the day,it is more convenient to use average daily totals for various latitudes and seasons as presented in Table 5.

The effect of clouds is to reduce the daily total solar radiation received at the sea surface, although a momentary increase may be effected when the sun is unobscured but surrounded by clouds. With an overcast sky the amount of reduction depends on the density of the clouds; it can be small with a thin cirrostratus cover, and with cumulonimbus the radiation can be reduced to one per cent. A useful expression for the ratio of the solar radiation received at the sea surface under average conditions to the radiation with cloudless

sky is

$$Q/Q_o = 1 - 0.71\ C,$$

where C is the proportion of the sky covered by clouds.+

Of the solar radiation reaching the sea surface most is absorbed in the water, but a small part is reflected at the surface or scattered back after entering the water. The proportion so lost, or albedo of the sea surface, depends on the altitude of the sun, on the cloudiness, and on the roughness of the surface. In clear weather the albedo increases with decreasing altitude of the sun. A rough surface increases the albedo slightly when the sun is in the zenith and decreases it considerably when the sun's altitude is low.

For overcast sky the albedo of the sea surface is always about 8 per cent. For clear sky the albedo is 3 to 6 per cent when the sun's altitude is greater than 30°, and increases to 20 per cent as the altitude decreases from 30° to 12°. For clear sky the albedo increases still further as the altitude falls below 12° if the sea is glassy, but it is probably never more than 20 per cent when the sea is rough.++

The average effective albedo throughout the day is therefore 8 per cent when the sky is overcast. At the equinoxes in clear weather with rough water it is about 6 per cent at the equator, 7 per cent at latitude 40°, 9 per cent at latitude 60°. In clear weather with rough water a maximum average albedo of about 20 per cent is reached in winter in high latitudes where the noon altitude of the sun is not more than 12°.

The solar energy absorbed in a day by one square centimeter of

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+Kimball, Monthly Weather Review, Vol. 56, page 394, 1929.
++Powell and Clarke, Journal of the Optical Society of America, Vol.26, pp. 111-120, 1936.

the ocean may thus be estimated by first taking the appropriate value from Table 5. This is then reduced for the average cloudiness during the hours the sun is high in the sky. The resulting value may then be corrected for the albedo of the sea surface.

Nocturnal radiation

Since the ocean is very nearly an ideal radiator the only difficulty in estimating nocturnal radiation is in finding the radiation from the atmosphere. Water vapor and clouds are by far the most important absorbers and radiators of long-wave radiation in the atmosphere. In clear weather, therefore, the long-wave radiation received at the sea surface depends on the vertical distribution of water vapor and of temperature in the atmosphere.

From surface observations there is no direct means of estimating these distributions. It has however been found that vapor pressure at deck level serves as an approximate index of the effect of these distributions on nocturnal radiation. Table 6 has been computed to give the nocturnal radiation as a function of sea-surface temperature and of vapor pressure at deck level. In addition the emission of an ideal radiator is shown for each temperature.

The nocturnal radiation is reduced by clouds. Their effect depends on their temperature and density. To obtain a rough evaluation of nocturnal radiation with overcast skies one may take the following percentages of the appropriate value in Table 6: 75 per cent for cirrostratus, 35 per cent for altostratus, 15 per cent for stratus or numbostratus.

Mixing in the air above the sea surface

Next to the sea surface the vapor pressure has its equilibrium

value with respect to the sea water, and the air has the same temperature as the water. In the air near the surface there is generally a vertical gradient of both vapor pressure and temperature. In a region of such gradients any turbulence or mixing which may be present transports water vapor vertically from levels of higher vapor pressure toward levels of lower vapor pressure, and transports heat from warmer levels toward colder levels. This process is easily understood qualitatively, because mixing consists of a rather random vertical motion in which of necessity as much air moves upward as moves downward through a large horizontal area. If temperature decreases upward for instance, the turbulent parcels moving up through this area are on the average warmer than those moving down, so the net transfer of heat is upward.

This vertical transport by mixing brings heat and water vapor from or to the sea surface, and thus accounts for heat conduction and evaporation or condensation at the surface. The heat and water vapor lost, or gained, by the surface are not to any extent supplied to, or by, the lowest layers of air, but are distributed by mixing through a deep layer of air. The vertical transport is therefore practically the same at all levels sufficiently near the surface -- certainly for all levels up to any masthead.

The rate of vertical transport of heat or water vapor is proportional, for unit horizontal area, to the product of the gradient of temperature or vapor pressure and of the intensity of mixing. Near a boundary such as the sea surface the intensity of mixing increases with distance from the boundary. Since the intensity of mixing increases upward from the sea surface, and since the vertical transport of heat or vapor is the same at all levels near the surface, it follows that the gradients of temperature and water vapor decrease

upward.

The vapor pressure at deck level is usually less than at the sea surface. In this case the average vertical distribution of vapor pressure is seen to be a continuous decrease from the sea surface to deck level and beyond, the decrease being very steep at the surface. Actually the change of vapor pressure or temperature from the surface to an elevation a centimeter or two above is about as much as the change from the latter to deck level. The gradient at deck level is already so slight that the elevation of deck level has little effect on temperature or humidity measurements.

An instantaneous vertical distribution has no such regular shape, and at any fixed level temperature and humidity usually fluctuate rapidly. This is often noticed in using a psychrometer, and is simply a manifestation of the irregular turbulent motion.

The intensity of mixing varies with elevation above the surface, and at a given elevation depends on the wind force, on the stability of the air, and on the roughness characteristics of the surface. The last two factors are fortunately usually unimportant for elevations sufficiently low over the open ocean. Hence the intensity of mixing can for practical purposes be expressed in terms of the wind force.

Wind force is in most ways preferable in this work to wind speed. The degree of turbulent mixing in the air is intimately associated with the shearing stress between the air and water, and the latter is directly evidenced in the appearance of the sea surface. Hence the wind force estimated from the appearance of the sea according to the instructions in Circular M of the United States Weather Bureau is excellently suited as an indication of turbulent mixing. It may be noted that such a scale based on the appearance of the surface is an

indication of the wind force on the horizontal boundary of the atmosphere , not of the wind force on a vertical wall. It is a boundary phenomenon, and there can be no meaning in speaking of its variation with elevation above the surface. Wind force is easily observed, although there is the disadvantage that gradations finer than Beaufort units or half Beaufort units cannot be distinguished reliably. On the other hand wind speed varies with elevation above the surface, and for a given wind force the speed at any level varies with the stability of the air. Furthermore wind speed cannot be measured accurately on shipboard. Of course the air speed past an anemometer can be measured with sufficient accuracy, but even a good anemometer exposure on a stationary ship cannot be relied upon to give the wind speed at the same elevation free from the ship's influence. For a ship under way the difficulties are multiplied, so that no matter how carefully corrections are applied the wind speed derived from anemometer readings is not as reliable as the wind force estimated by an able observer. Hence, both theoretically and practically, it is justified to employ wind force.

To summarize the conduction of heat from the sea surface, this is equal to the vertical transport of heat by turbulence at any low level in the air. This turbulent transport at a chosen level is given as the intensity of mixing at that level times the temperature gradient at that level. The intensity of mixing depends on the wind force. The temperature gradient is proportional to the difference between the water temperature and the air temperature at deck level. Hence the rate of conduction may be expressed as the product of the water-air temperature difference and of a factor depending on wind force. This factor is tabulated in the next section. The rate of evaporation is found from a similar product as presented in the following section.

The basic research on turbulence in the atmosphere is still in

a preliminary stage. It is difficult because so far such quantities as mixing intensity and evaporation and heat conduction by turbulence have not been measured directly, quantitative information about them being always derived indirectly. Hence the numerical values in what follows are very uncertain, although there is every reason to believe that the order of magnitude is correct in each case.+

Conduction of heat between ocean and atmosphere

If t_s is the temperature of the surface water and t is the air temperature at deck level, the rate at which unit area of the sea surface loses heat to the atmosphere by conduction is

$$H_c = h_c (t_s - t).$$

Within the limitations already stated the values of the factor of h_c for various wind forces are:

Wind force, Beaufort	1	2	3	4	5	6
h_c, cal cm^{-2} day^{-1} (deg F)$^{-1}$	~~24~~ 2	~~47~~ 5	~~72~~ 7	~~100~~ 10	~~128~~ 13	~~158~~ 16

Evaporation

Let e_s be the vapor pressure at the sea surface (determined from surface water temperature and salinity by use of Table 2), and e be the vapor pressure at deck level (determined from psychrometer readings and barometric pressure by use of Tables 1 and 3). The rate of evaporation, expressed in terms of the thickness of the water layer evaporated in unit time, is

$$E = c (e_s - e),$$

where c has the values:

+ The chief source is Montgomery, Papers in Physical Oceanography and Meteorology, Vol. VII, No. 4, 1940.

Wind force, Beaufort	1	2	3	4	5	6
c, cm mb^{-1} day^{-1}	0.012	0.023	0.041	0.072	0.122	0.190

(In estimating these values for c the evaporation from spray has been included along with the evaporation from the sea surface itself.)

Due to the latent heat withdrawn in vaporization, for every centimeter of water evaporated about 590 cal cm^{-2} of heat is utilized, so that water is cooled by evaporation at the rate

$$H_e = h_e (e_s - e).$$

The factor h_e has the values:

Wind force, Beaufort	1	2	3	4	5	6
h_e, cal mb^{-1} cm^{-2} day^{-1}	7	14	24	42	72	112

Evaporation not only cools the surface water but also increases its salinity, both of which increase its density. Except at low temperatures the density is more affected by the cooling than by the increase of salt concentration. Since the other factors producing a change in density all involve heat it is convenient to replace this salinity increase by an equivalent heat loss. Thus the effect on density of the salinity increase may be represented by a fictitious heat loss

$$H_s = k H_e,$$

where k has the values (for salinity 35 per mille):

Water temperature, °F	30	40	50	60	70	80	90
k	1.6	0.44	0.29	0.22	0.18	0.15	0.13

As far as density is concerned, evaporation leads to a total effective heat loss

$$H_e + H_s = H_e (1 + k).$$

Summation

Let S be the rate at which solar radiation is absorbed by the

ocean per unit area, and let N be the nocturnal radiation. Then the net rate at which unit area of the ocean gains heat at the surface is

$$S - N - H_c - H_e.$$

Of the individual terms, S is always positive, N and H_e are usually positive, H_c can often be negative. Of course the sum is sometimes positive and sometimes negative.

Omitting from consideration a thin surface film, the density of the surface water is being increased if

$$S - N - H_c - H_e (1 + k)$$

is negative, and it is being decreased if this sum is positive.

The unit of area used in the calculations is the square centimeter. The unit of heat is the calorie, which is the heat used in warming one gram of water by one degree centigrade; 102 cal cm^{-2} will warm one fathom of water by one degree Fahrenheit. The unit of time is the day. This was chosen because it is much simpler to work with daily totals of solar radiation than with instantaneous values. It is usually a simple matter to obtain results for periods shorter than a day. If each quantity is expressed in terms the day as time unit, the total loss of heat during a 12-hour night is for instance

$$1/2 (N + H_c + H_e),$$

and the gain of heat during a 12-hour day is

$$S - 1/2 (N + H_c + H_e).$$

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TABLE 1

Vapor pressure over water in millibars

Temp. °F	0	1	2	3	4	5	6	7	8	9
10	2.4	2.5	2.6	2.7	2.9	3.0	3.1	3.3	3.4	3.6
20	3.7	3.9	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.4
30	5.6	5.9	6.1	6.4	6.6	6.9	7.2	7.5	7.8	8.1
40	8.4	8.7	9.1	9.4	9.8	10.2	10.6	11.0	11.4	11.8
50	12.3	12.7	13.2	13.7	14.2	14.8	15.3	15.9	16.5	17.1
60	17.7	18.3	19.0	19.7	20.4	21.1	21.8	22.6	23.4	24.2
70	25.1	25.9	26.8	27.7	28.7	29.7	30.7	31.7	32.8	33.9
80	35.0	36.2	37.3	38.6	39.8	41.1	42.5	43.9	45.3	46.7
90	48.2	49.7	51.3	52.9	54.6	56.3	58.1	59.9	61.7	63.6

Converted from Smithsonian Meteorological Table No. 75 and from Harrison, Monthly Weather Review, Vol. 62, pp. 247-248, 1934.

TABLE 2

Vapor pressure over sea water of salinity 35 per mille, in millibars. The salinity effect is linear with salinity; correction to be added for salinity 25 per mille is given in first column for temperature in the second column.

Corr'n for 25 °/oo	Temp. °F	0	1	2	3	4	5	6	7	8	9
+0.01	10	2.4	2.5	2.6	2.7	2.8	2.9	3.1	3.2	3.3	3.4
+0.02	20	3.6	3.8	4.0	4.1	4.3	4.5	4.7	4.9	5.1	5.3
+0.03	30	5.5	5.8	6.0	6.2	6.5	6.8	7.0	7.3	7.6	7.9
+0.05	40	8.2	8.6	8.9	9.2	9.6	10.0	10.4	10.8	11.2	11.6
+0.07	50	12.0	12.5	13.0	13.5	14.0	14.5	15.0	15.6	16.1	16.7
+0.09	60	17.3	18.0	18.6	19.3	20.0	20.7	21.4	22.2	23.0	23.8
+0.13	70	24.6	25.4	26.3	27.2	28.1	29.1	30.1	31.1	32.2	33.2
+0.19	80	34.3	35.5	36.6	37.9	39.1	40.4	41.7	43.0	44.4	45.8
+0.26	90	47.3	48.8	50.3	51.9	53.6	55.2	57.0	58.7	60.5	62.4

Computed from the equation

(vapor pressure over salt water) =
(vapor pressure over water)(1 - 0.537 x salinity).

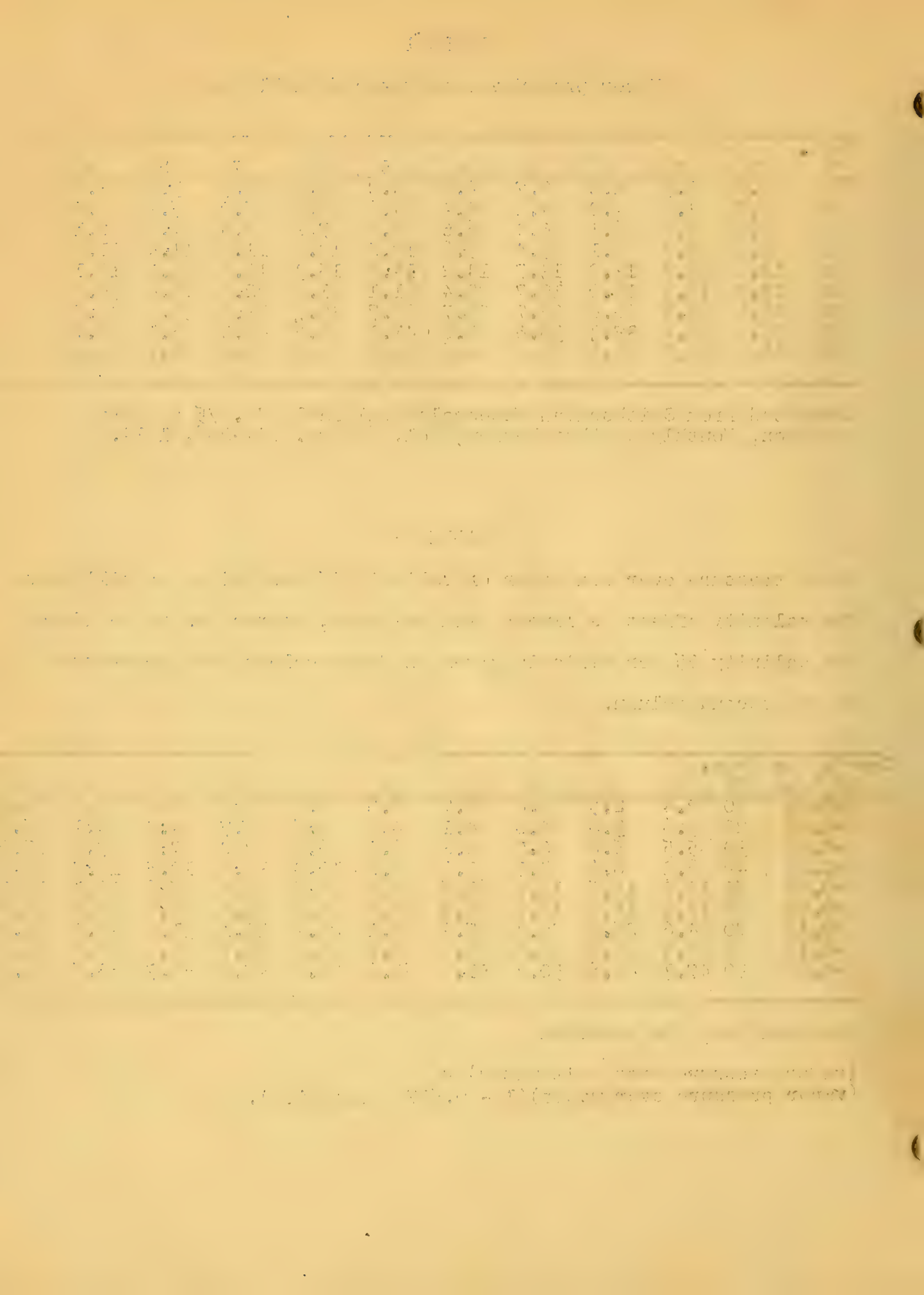

TABLE 3

Vapor pressure from psychrometer readings

This formula applies if the ventilation speed is 3 meters per second or more.

t : air temperature

t' : temperature of the wet bulb

e : vapor pressure

e' : saturation vapor pressure for the temperature of the wet bulb

$$e = e' - F(t-t')$$

The factor F varies slightly with the temperature of the wet bulb and with the barometric pressure. The following table gives F in millibars per degree Fahrenheit:

t', °F	32	40	50	60	70	80	90
980 mb	0.360	0.361	0.364	0.366	0.368	0.371	0.373
990 mb	0.363	0.365	0.368	0.370	0.372	0.374	0.377
1000 mb	0.367	0.369	0.371	0.374	0.376	0.378	0.381
1010 mb	0.371	0.373	0.375	0.377	0.380	0.382	0.384
1020 mb	0.374	0.376	0.379	0.381	0.383	0.386	0.388
1030 mb	0.378	0.380	0.382	0.385	0.387	0.390	0.392
1040 mb	0.382	0.384	0.386	0.389	0.391	0.393	0.396
29 in.	0.360	0.362	0.365	0.367	0.369	0.372	0.374
30 in.	0.373	0.375	0.377	0.380	0.382	0.384	0.387
31 in.	0.385	0.387	0.390	0.392	0.395	0.397	0.400

These values were computed from the formula on page lxx of the Smithsonian Meteorological Tables, Fifth revised edition, giving, for temperature Fahrenheit;

$$F = 0.000367\ B\ (1 + 0.00064(t' - 32°)),$$

where B is the barometric pressure.

TABLE 4

Instantaneous values of solar radiation received on unit area of the sea surface in the absence of clouds. Values are given for the two extreme values of water vapor content in the atmosphere likely to occur commonly, namely 0.5 cm and 5 cm of precipitable water.

Solar altitude	z	cal cm^{-2} day^{-1} 0.5 cm	cal cm^{-2} day^{-1} 5 cm	cal cm^{-2} hour^{-1} 0.5 cm	cal cm^{-2} hour^{-1} 5 cm
90°	0	2320	1880	97	78
80°	10	2290	1877	95	77
70°	20	2170	1750	90	73
60°	30	1980	1580	82	66
50°	40	1720	1350	72	56
40°	50	1400	1070	58	44
30°	60	1040	756	43	31
20°	70	660	429	27	18
10°	80	278	127	12	5

Computed using 2800 cal cm^{-2} day^{-1} as the solar constant and the atmospheric transmissions given by Kimball (Fig. 1, Monthly Weather Review, Vol. 56, pp. 393-398, 1929).

TABLES 5

Average daily total solar radiation on the 21st of each month received on unit area of the sea surface in the absence of clouds, in calories per square centimeter. The values include both the direct solar beam and the scattered radiation from the sky.

Northern hemisphere	Dec.	Jan.	Feb.	March	April	May	June
Southern hemisphere	June	July	Aug.	Sept.	Oct.	Nov.	
		May	April	March	Feb.	Jan.	Dec.
Latitude							
60°	13	49	210	388	611	799	837
50°	141	187	324	480	652	790	858
40°	254	295	413	563	704	794	838
30°	365	401	491	613	714	800	822
20°	468	488	587	672	721	759	784
10°	533	562	634	702	721	711	708
0°	630	641	678	685	678	641	630

Obtained by averaging and interpolating Kimball's values (Table 3, Monthly Weather Review, Vol. 56, pp. 393-398, 1929).

TABLE 6

Nocturnal radiation with clear sky, in calories per square centimeter per day.

Water temp., °F	30	40	50	60	70	80	90
Ideal radiator	644	698	756	817	881	950	1022
Vapor pressure, mb							
2	288	312	338	365	394	425	457
4	258	279	302	327	352	380	409
6	234	254	275	297	321	346	372
8	215	233	252	273	294	317	341
10	198	214	232	251	270	292	314
15		175	189	204	220	238	256
20			153	165	178	192	207
25				131	141	152	164
30					107	116	125

Nocturnal radiation for clear sky computed from

$$N_c = sT^4 (1 - a - b\sqrt{e})$$

using $a = 0.44$ and $b = 0.08$ $mb^{-1/2}$ (Brunt, Physical and dynamical meteorology, Second edition, pp. 136-137), and $s = 1.178 \times 10^{-7}$ cal cm^{-2} day^{-1} $(deg\ A)^{-4}$.

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Nocturnal radiation with clear sky, in calories per square centimeter per day.

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Ideal radiator	644	698	756	817	883	950	1022
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8	215	233	252	273	294	317	341
10	198	214	232	251	270	292	314
15		175	189	204	220	238	256
20			153	165	178	192	207
25				131	141	152	164
30					107	116	125

Nocturnal radiation for clear sky computed from

$$N_o = \sigma T_a^4 (1 - a - b\sqrt{e})$$

using $a = 0.44$ and $b = 0.08$ mb$^{-1/2}$ (Brunt, Physical and dynamical meteorology, Second edition, pp. 136-137), and $\sigma = 1.178 \times 10^{-7}$ cal cm^{-2} day^{-1} (deg A)$^{-4}$.